

Math 3215: Homework 1 Answers

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1 Sample spaces

We want to model the roll of a green and a red 6-sided dice.

1. What is the most natural sample space for this experiment? (You don't need to list all the outcomes; either write the sample space formally in math language, or describe the sample space and give examples of individual outcomes).
2. Is 'The red die comes up 6' an outcome in this sample space?
3. Is it an event?
4. Come up with a sample space in which 'The two dice come up the same' is an *outcome*.
5. Is this sample space a *refinement* of the natural sample space?
6. Write a question you could ask in the first sample space but not the second.

Answers:

1. The set of pairs like $(1, 5)$ with the first being the roll of the green die and the second the roll of the red. There are 36 outcomes in this sample space.
2. No, it's not.
3. It's an event since it's a collection of six outcomes : $\{(1, 6), (2, 6) \dots (6, 6)\}$.
4. $S = \{\text{dicethesame}, \text{dicedifferent}\}$.
5. No, it's much coarser.
6. Was the sum of the dice greater than 7?

2 Sample spaces

Consider modeling the opening-day ticket sales of the new Batman movie.

1. Describe a sample space for this model that you think is too coarse.
2. Describe a sample space you think is too fine.
3. Describe a sample space you think is appropriate.

Answers:

1. $S = \{\text{Themoviemade} > \$100, \text{themoviemade} \leq \$100\}$
2. Each outcome would be a complete specification of which people in America bought or did not buy a ticket.
3. An outcome would specify the exact number of tickets sold in each city in America.

3 Set theory

Let A, B and C be events. Write the following events in set-theoretic language and draw a Venn diagram with each event below labeled. Example: A, B and C all occur: $A \cap B \cap C$.

1. At least two of the three events (A, B, C) occur.
2. At most one of the three events occurs.
3. A and B occur, but not C
4. Either C and B occur or A does not occur

Answers:

1. $(A \cap B) \cup (A \cap C) \cup (B \cap C)$
2. $((A \cap B) \cup (A \cap C) \cup (B \cap C))^c$
3. $A \cap B \cap C^c$
4. $(B \cap C) \cup A^c$

4 Equally likely outcomes

Consider the experiment of shuffling a deck of 52 cards, then drawing two cards off the top. The natural sample space has $52 \cdot 51$ equally likely outcomes.

Answer the following *using the formula* $\Pr(E) = \frac{|E|}{|S|}$.

1. What is the probability of the first card being red?
2. What is the probability of drawing a King and Queen?
3. What is the probability of drawing two red cards?
4. What is the probability of drawing no 2's or 3's?

Answers:

1. $\frac{1}{2}$
2. $\frac{2}{13 \cdot 13}$
3. $\frac{26 \cdot 25}{52 \cdot 51}$
4. $\frac{44 \cdot 43}{52 \cdot 51}$

5 Set Theory

We've seen a Venn diagram that shows all possible unions and intersections of 3 sets, A, B, C .

1. Can you draw a Venn diagram that shows all possible intersections of 4 sets? Try to make the drawing as clear and easy to understand as possible
2. Is it possible to draw such a diagram for 20 sets? If so, describe how (don't actually draw it), if not, say why not.

6 Countable infinite sample space

Let $S = \{0, 1, 2, \dots\}$, and $P(i) = C \cdot \left(\frac{2}{3}\right)^i$.

1. What is C ? (the actual number)
2. What is the probability of an even number? i.e. $\Pr[\{0, 2, 4, \dots\}]$.

Answers:

1. We know that $\Pr[S] = 1$, so $C = \frac{1}{\sum_{i=0}^{\infty} (2/3)^i} = \frac{1}{3}$ (a geometric series)
2. One way to do it: $P(1) = \frac{2}{3}P(0)$, $P(3) = \frac{2}{3}P(2)$ and so on, so $P(\text{odd}) = \frac{2}{3}P(\text{even})$ and so

$$1 = P(\text{even}) + P(\text{odd}) = \frac{5}{3}P(\text{even})$$

So $P(\text{even}) = \frac{3}{5}$.