

# Math 3215: Homework 2

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## 1 Counting

- Two poker players each have gotten 4 cards. Player  $A$  has the 2, 3, 5, 6 of Clubs, while player  $B$  has the 5, 6 of hearts and 7, 8 of diamonds. They each get one more card from the deck
  - What is the probability player  $A$  gets a straight?
  - Player  $B$ ?
  - What is the probability player  $A$  gets a straight but player  $B$  doesn't?
  - What is the probability player  $A$  gets a flush?
  - Is the event that player  $A$  gets a straight independent from the event that player  $B$  gets a straight?
  - Is the event that player  $A$  gets a straight independent of the event that player  $A$  gets a flush?
  - Is the event that player  $A$  gets a flush independent of the event that player  $B$  gets a straight?
- How many 10-digit sequences are non-decreasing? (e.g. the sequence  $\{0, 1, 1, 3, 4, 5, 6, 8, 8, 9\}$  is one such sequence).

## 2 Conditional Probability

- Let  $A$  be the event that it rains tomorrow,  $B$  the event that a hot-dog man makes at least \$100, and  $C$  the event that he returns home happy. Say  $\Pr[A] = .2$ ,  $\Pr[B] = .7$  and  $\Pr[C] = .6$ .
  - Draw a Venn diagram with all three events and label each different region (there are 8) with the corresponding probabilities in such a way that  $A, B, C$  are jointly *independent*.
  - Draw and label a new Venn diagram, keeping  $\Pr[A], \Pr[B], \Pr[C]$  the same, but labeling the 8 regions with probabilities in such a way that  $A, B, C$  are not jointly independent and in fact no pair is independent.
- 60% of Georgia Tech students are from Georgia. 20% of students from Georgia attend a football game, but only 10% of students from out-of-state attend. If you pick a random student at the football game, what is the chance he or she is from Georgia?
- You throw your business card in a jar at your favorite restaurant. At the end of the month, there are 200 cards in the jar, and they choose cards out at random (without replacement) and give the winners a free lunch.
  - What's the probability your card is drawn out first?
  - What's the probability your card is drawn out fifth?
  - What's the probability your card is drawn somewhere in the first five draws?
  - Once you've seen that your card wasn't drawn out first, what's the probability it will be drawn out fifth?

### 3 Independence

1. Isaac Newton's Problem: You have a bunch of fair dice. Rank the following three events most to least likely:
  - You throw at least one 6 rolling 6 dice.
  - You throw at least 2 6's rolling 12 dice.
  - You throw at least 3 6's rolling 18 dice.
2. 60% of South Carolinians prefer Newt Gingrich and 40% prefer Mitt Romney. If we pick 7 South Carolinians at random and ask their preference, what's the chance that more prefer Romney than Gingrich?
3. We take a survey of 500 people. We ask three questions,  $A$ ,  $B$ , and  $C$ , all yes no questions. We get the following set of responses:
  - $\{y, y, y\}$ : 49 people
  - $\{y, y, n\}$ : 44 people
  - $\{y, n, y\}$ : 3 people
  - $\{y, n, n\}$ : 6 people
  - $\{n, y, y\}$ : 174 people
  - $\{n, y, n\}$ : 184 people
  - $\{n, n, y\}$ : 23 people
  - $\{n, n, n\}$ : 17 people
  - (a) Do you think the people's opinions on the three different issues are independent? Explain in detail.
  - (b) Can you think of a method to test whether or not the opinions are independent of each other?
  - (c) Name two opinions Americans might have that are probably not independent of each other.
  - (d) Name two opinions Americans might have that probably are (close to) independent of each other.
4. Prove: If  $A$  is independent of  $B$  then  $A^c$  is independent of  $B$  and  $A^c$  is independent of  $B^c$ . (use the definition of independence)