Math 3215: Homework 5

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due March 29

Worth 10 homework points instead of 5.

Note: For several of these problems, you will need to use probabilities of a normal random variable. Look up 'table of normal distribution' on the internet to find these, or use a computer program like mathematica. Also look up probabilities of the t-distribution when needed.

1 Parameter Estimation and Confidence Intervals

1. Should a 95% confidence interval be narrower or wider than a 99% confidence interval for the same parameter?

2. You poll 400 people and 60% of them say that will vote for Romney next fall. What is a 95% confidence interval for the true proportion?

3. You take a sample $X_1, \ldots X_{100}$ of iid Poisson random variables with unknown mean λ. If $\sum_1^{100} X_i = 200$, what is a 99% confidence interval for λ?

4. You take a sample of 100 students and find their average height is 6 ft, and the sample variance $s^2 = .5$. What is a 90% confidence interval for the true average height of a student?

2 A Spreadsheet

Make an excel (or google documents) spreadsheet using formulas that does the following:

- The user inputs a 'margin of error', something like .03 or .1
- The user inputs a confidence level, something like .95 or .99.
- the formula in the spreadsheet calculates n so that if a poller takes a sample size of n in a political poll, with probability at least the confidence level, the actual proportion of voters will be within the margin of error of the sample mean.
- for the variance assume the worst case, i.e. $p = .5$.
- Email me this spreadsheet.

3 Real-world practice

- Find a data set on the internet. It could be sports stats from espn, demographic data, election data, weather data, anything you like. It should be raw data.
- Copy it into a spreadsheet.

- Come up with an unknown parameter and give an estimate and a 95% confidence interval for this parameter.
- Email me a link to the data set and your spreadsheet along with the one above.

4 Central Limit Theorem

1. Write down the conditions for the CLT to be true. Looking at the proof of the CLT which of these conditions do ou think can be relaxed?
2. Is there such thing as a random variable X with infinite variance? Can you construct a discrete random variable X with infinite variance?

5 Maximum Likelihood

1. Assume $X_1, \ldots X_n$ are i.i.d. discrte random variables which take the value i with probability $p^i(1-p)$, $p = 0, 1, \ldots$. Here p is an unknown parameter.
   - What is the maximum likelihood estimator for p? Prove it
   - Is the MLE an unbiased estimator of p?
   - What is the variance of the MLE?
2. Let $X_1, \ldots X_n$ be i.i.d. and each take the values $-1, 0, 1$ with probabilities $1/2-p/2, p, 1/2-p/2$ respectively, where p is an unknown parameter.
   - What is the MLE for p?
   - What is the mean and variance of the MLE?
3. Let $Y_1, \ldots Y_n$ be i.i.d. Normal random variables with mean 0 and unknown variance σ^2.
   - Find the MLE estimator for σ^2
   - Is the MLE unbiased?
4. Let $P_1, \ldots P_n$ be i.i.d. Normal random variables with unknown mean λ.
   - Find the MLE estimator for λ
   - Is the MLE unbiased?
   - The mean and variance of a Poisson are the same. Does this give another estimator (in addition to the MLE) for λ? Is this estimator unbiased?