Math 3215: Lecture 13

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1 Joint CDF's and Joint Density Functions

Another perspective: Imagine random variables X and Y each take the values $\{1, \ldots n\}$ (they're discrete rv's). We can write down a matrix A where $A_{ij} = \Pr[X = i \cap Y = j]$.

- Joint CDF: $F_{X,Y}(s,t) = \Pr[X \leq s \cap Y \leq t]$. We can shade the area on the matrix this represents (lower left corner).
- Joint density function roughly corresponds to the i, jth entry of A.

2 Marginal CDF's and Density Functions

How to calculate marginal distributions:

- Marginal just means 'by itself'
- The marginal CDF of X is just $F_X(t) = \Pr[X \leq t]$
- in the matrix example $F_X(t)$ corresponds to summing over the lower portion of the matrix. $F_Y(t)$ would correspond to summing over the left portion of the matrix.

Questions:

1. Let X be uniform on $[0, 2]$ and $Y = 2X + 1$. What is the joint density function? The joint CDF? The marginals?
2. Suppose the joint density function $f(x, y)$ factors into two functions, one that depends only on x and the other that depends only on y, i.e. $f(x, y) = g(x) \cdot h(y)$. Prove that X and Y are indepedent random variables.
3. Let $f(x, y) = \frac{3}{28}\left(x^2 + y\right)$ for $x, y \in [0, 2]$ and 0 elsewhere. Is this a valid joint density function? (if not, fix it!) Calculate:
    - The joint CDF
    - The marginal density functions
    - The marginal CDF's
    - What is $\Pr[Y \geq 1 | X \geq 1]$?

3 The Normal Distribution

(we'll be using this more and more so it's a good idea to get comfortable with it).

X is a standard normal r.v. if its density function is $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

- Why the factor in front? I'll show the integration on the board.
- What is the mean and variance of X?
- If $Y = 2X + 1$, what is the mean and variance of Y?
- in general if $Y = aX + b$, what is the mean and variance of Y?

4 Buffon's Needle

A Frenchman named Buffon came up with this problem in the 1700's. It will test your understanding of continuous random variables.

We have a wood floor with parallel wood floorboards running along it. The wood planks are 1 inch wide (and very long, let's say). We have a needle, also 1 inch wide. If we toss the needle up in the air at random, what is the chance that it lands crossing the crack between two floorboards?

Solve it!

Some steps:

1. Make a rough guess to check the sanity of your later answer.
2. Come up with a good model.
3. Write the model mathematically.
4. Describe mathematically the quantity you want to solve for.
5. Do the calculations.