

# Math 3215: Lecture 2

Will Perkins

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## 1 Set Theory Basics

Events are sets of outcomes, and subsets of the entire sample space. In what follows,  $A$  and  $B$  are sets. We can write  $A \subseteq S$  and  $B \subseteq S$ . To illustrate, let's think of  $A$  as the event that it rains tomorrow,  $B$  the event that it is cold, and  $C$  the event that it is sunny tomorrow.

Three basic operations on sets:

1. Union.  $A \cup B$ . It is rainy *or* cold.
2. Intersection.  $A \cap B$ . It is rainy *and* cold.
3. Complement.  $A^c$ . It is *not* rainy.

Vizualizing sets and set operations: Venn Diagrams.

Draw and describe in word the following events:

- $(A \cup B)^c$
- $(A \cap B)^c$
- $A^c \cap B^c$
- $A \cap (B \cup C)$
- $(A \cup B) \cap (C \cup B)$

## 2 Axioms of Probability

These axioms hold for all probability models, discrete and continuous.

1.  $P(A) \in [0, 1]$  for all events  $A \subseteq S$
2.  $P(S) = 1$
3.  $P(A \cup B) = P(A) + P(B)$  if  $A \cap B = \emptyset$
4.  $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$  if  $E_i \cap E_j = \emptyset$  for all  $i \neq j$ .

## 3 DeMorgan's Laws

de Morgan's Laws are:

$$(A \cup B)^c = A^c \cap B^c$$

and

$$(A \cap B)^c = A^c \cup B^c$$

## 4 Inclusion / Exclusion

The inclusion / exclusion principle is:

$$\Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B]$$

- Prove this using the Axioms of Probability
- Generalize it. What is  $\Pr[A \cup B \cup C]$ ?
- Can you generalize it to  $k$  sets?

## 5 Equally likely outcomes

Sometimes each outcome in a sample space has the same probability. Think flipping a coin, or rolling a die, or picking a student from class at random, or picking two students from class at random.

In this case, the probability of an event can be computed by counting.

$$\Pr[A] = \frac{|A|}{|S|}$$

where  $|A|$  is the number of outcomes in  $A$  and  $|S|$  is the number of outcomes in the entire sample space.

## 6 Questions

1. If I tell you that the probability that it is warm is .2 and the probability that it is sunny is .4, can you tell me the probability that it is sunny and warm?
2. Kobe Bryant goes to the line for two free throws. Describe a full probability model for what happens, with a sample space and a probability function. Explain why you chose each.
3. Give an example where equally likely outcomes make sense, and an example where they do not.
4. Can a sample space have an infinite number of outcomes, each of which has a positive ( $> 0$ ) probability? If so give an example.
5. If I pick two students out of a class of 30 at random, how many outcomes are possible?
6. If there are 4 students in the front row, what is the probability that both students I pick come from the front row?