Math 3215: Lecture 20

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1 Chi Square distribution

Let $Z_1, \dots Z_k$ be independent $N(0,1)$ random variables. Then $Q = \sum_{i=1}^k Z_i^2$ has a *Chi Square* distribution with k 'degrees of freedom'.

Characteristic function for $Q_k : \phi_{Q_k}(t) = (1-2it)^{-k/2}$.

Questions:

- What is the mean of Q_k?
- Can you guess the variance of Q_k?
- What are the minimum and maximum possible values of Q_k?
- What can you say about the distribution of Q_n as $n \to \infty$?

2 Confidence Intervals for Variance

Recall that

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \overline{X})^2$$

is an unbiased estimator for σ^2. Now say we know that each X_i comes from a Normal distribution with unknown mean μ and unknown variance σ^2. How can we get a confidence interval for σ^2?

The steps (and these apply to creating confidence intervals for any parameter)

1. Understand the distribution (or approximate distribution) of your estimator
2. Find values a and b so that $\Pr[a \le E \le b] = .95$ (or whatever confidence you want)

Exercise:

- Find a normalization of S^2 so that you know its distribution.
- Use this normaliztion to find a 95% confidence interval for σ^2.

To begin the first exercise, start with the following:

$$Y = \frac{1}{\sigma^2} \sum_{i=1}^n (X_i - \mu)^2$$

We know from the definition of the Chi Square distribution that $Y \sim \chi_n$.

Now use the following expansion of Y:

$$Y = \frac{1}{\sigma^2} \sum_{i=1}^n (X_i - \overline{X} + \overline{X} - \mu)^2$$