Math 3215: Lecture 20

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Reminder: Quiz on Thursday for 10 midterm points.

1 Back to Hypothesis Testing

Now that we know something about the Central Limit Theorem and Normal approximation, we can revisit hypothesis testing.

Example 1: You run a factory making nails and you want an error rate of less than .01 (i.e. only 1 out of 100 nails can be a dud on average). An engineer comes up with a new manufacturing process and you test it on 900 nails, and find it only produces 7 duds. Should you use the new process or test this process more? Testing is expensive,but not as expensive as having to halt manufacturing if the process has too high an error rate. What should you do?

How to think about a problem like this:

- What kind of answer do we want? A strict yes/no or something quantitative?
- What probability distribution might be appropriate?
- Once we have a distribution in mind, what kind of calculation should we make?

2 Testing Equality of Two Means

Example: You're a farmer and you want to test two different brands of fertilizer to see which one helps your corn crow more. You treat 400 stalks of corn with Brand 1 and 400 with Brand 2. The average growth in the first sample is 10.4 inches and in the second is 9.4 inches. You compute $S_1^2 = 1$ and $S_2^2 = 1$. What can you conclude?

3 Goodness of Fit Testing

Say we don't know what type of distribution our random data is coming from, but we have a guess. How can we test whether or not we are right?

Example: We see a sequence of independent random digits from 1 to 6. We want to know if the distribution is uniform, i.e. like a fair die. We know the complete specification of the hypothesized distribution: $p_1 = p_2 = \cdots = p_6 = \frac{1}{6}$. Our observed data consists of $x_1, \ldots x_6$ where x_1 is the number of 1's that appeared and so on.

If the null hypothesis (the distribution is $\{p_i\}$) is true, then we would expect, as n gets large, for $\frac{x_i}{n}$ to be close to p_i for each i. How can we devise a test to measure this?

- Exercise: come up with a test (i.e. a statistic) and a method of calculating p-values that will tell us whether or not the true distribution is close to the hypothesized distribution

4 Goodness of Fit with Parameter Estimation

Here is a more complicated example:

You are a physicist and you use a Geiger counter to measure the number of radioactive particles emitted by a piece of plutonium in 10 milleseconds. You repeat the experiment and you see the following counts: $6, 2, 8, 6, 4, 4, 1, 9, 7, 6, 3, 5$.

How can you test whether or not the number of particles has a Poisson distribution? The difference between this and the last scenario is that here you do not have a complete specification of the hypothesized distribution, only its type.