Math 3215: Lecture 4

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1 Kahneman Tversky Problem

Blue and green taxi cabs, a witness with bad eyesight.

Conditional Probability

An important question: How do you update your probabilities / uncertanties / beliefs when you get new information? How do we do this is a consistent, logical manner?

Recall our interpretation of probability as your level of belief that an event will happen. If you learn new information, this level of belief can change. *Conditional probability* is a way of recalculating probabilities under new information.

Definition of Conditional Probability

Let A and B be events in the same probability model. The conditional probability of A given B is:

$$\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]}$$

if $P(B) \neq 0$.

We think of the original model restricted to the event B as a new probability model whose outcomes are just the outcomes belonging to B. The probability function has to change however, to make the total probability 1. So we scale by a factor of $\frac{1}{\Pr[B]}$. This is called *renormalizing*.

2 An urn model

You have an urn filled with r red balls and b black balls. You draw balls out at random, and do not replace them in the urn.

Questions:

1. Given that you draw a red ball first, what is the chance you draw a red ball second?
2. What is the probability that the first two balls drawn out are red?

3 A multiplication rule

$$\Pr[A \cap B] = \Pr[B] \cdot \Pr[A|B] = \Pr[A] \cdot \Pr[B|A]$$

more generally,

$$\Pr[E_1 \cap E_2 \cap \cdots \cap E_k] = \Pr[E_1] \cdot \Pr[E_2|E_1] \cdots \Pr[E_k|E_1 \cap \ldots E_{k-1}]$$

(Prove this using the definition of conditional probability).

4 Monte Hall Problem

3 doors, 2 goats, 1 Cadillac.

5 Questions

1. We flip a fair coin 5 times. What's the probability our first 3 flips are heads given that we get a total of exactly 3 heads? Given that we get exactly 4 heads?
2. What is the probability we get exactly 4 heads given that our first 3 flips are heads?
3. What is the probability we get at least 4 heads given that we get at least 3 heads?
4. What is the probability the first two cards drawn off a deck are diamonds given that the first card drawn is a diamond?
5. What is the probability the first two cards are diamonds given that the second is a diamond?
6. What is the probability the first two cards are diamonds given that neither is a club?
7. What is the probability a poker hand is a flush given that it is all red?
8. Is either of the following always true (a theorem):
   - $\Pr[A] \leq \Pr[A|B]$
   - $\Pr[A] \geq \Pr[A|B]$
9. If we roll two dice, what is the probability we roll at least one 5 given that we roll no sixes?
10. What is the probability we roll at least one 5 given that we roll at least one 6?
11. Write a formula for $\Pr[A|B]$ in terms of $\Pr[B|A]$.
12. If you know your poker hand is a flush (all 5 of the same suit) is it more likely or less likely that you have a straight than if you new your hand was not a flush? Or is the chance the same? Guess first, then solve.