

Math 3215: Lecture 5

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Today:

- Analyze the green/blue taxicab problem
- conditional probability and medical tests
- define independence

1 Bayes' Formula

Bayes' Formula is:

$$\Pr[A|B] = \frac{\Pr[A] \cdot \Pr[B|A]}{\Pr[B]}$$

- Prove Bayes' formula.

2 Medical Testing

Say Bayer has come up with a new diagnostic blood test for a certain rare disease. The disease affects one in two thousand people. If a person has the disease, the test correctly diagnoses it with probability .91. If the person does not have the disease, the test gives the correct answer (negative) with probability .99.

- Is this a good test?
- What should be the response of a doctor if her patient tests positive for this disease?
- Say a disease occurs in 1% of people. What must the false positive rate of a diagnostic test be so that we can be 90% sure that someone who tests positive does in fact have the disease?
- Am I being too simplistic in my description of diagnostic tests? Be specific.

3 Independence

Here is another important definition:

Two events, A and B , are *independent* if

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

What is your intuitive definition of independence? Do the two definitions agree with each other?

A set of k events $A_1 \dots A_k$ are *jointly independent* if the above multiplication rule holds for any subset of events:

$$\Pr[A_{i_1} \cap \dots \cap A_{i_j}] = \Pr[A_{i_1}] \cdots \Pr[A_{i_j}]$$

- Give an example of three events A, B, C s.t. any two of the events are independent, but the three events together are not independent.
- if A and B are independent, what is $\Pr[A|B]$?
- how do you draw two independent events on a Venn diagram? (careful, there is a common mistake here)

4 Questions

1. Is the event A independent of itself?
2. Is the event that we draw a red card second from a 52 card deck independent of the event that we draw a red card first?
3. Flip two fair coins. Is the event that the second coin a head independent of the event that the first coin is a head?
4. Is the event that the total number of heads odd independent of the event that the first coin is a head?
5. Shuffle a deck of cards. Let A be the event that the first diamond comes before the first heart, B be the event that the first club comes before the first spade, and C the event that the first diamond comes before the first club. Is A independent of B ? Is A independent of C ?