Math 3215: Lecture 6

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Today:

- Review definition of independence

- Independence of more than 2 events

- A biased coin

1 Independence

Definition: A and B are independent if $\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$.

Joint independence: A set of k events $A_1 \dots A_k$ are *jointly independent* if the above multiplication rule holds for any subset of events:

$$\Pr[A_{i_1} \cap \cdots \cap A_{i_j}] = \Pr[A_{i_1}] \cdots \Pr[A_{i_j}]$$

2 Questions

1. Is the event A independent of itself?

2. Is the event that we draw a red card second from a 52 card deck independent of the event that we draw a red card first?

3. Flip two fair coins. Is the event that the second coin a head independent of the event that the first coin is a head?

4. Is the event that the total number of heads odd independent of the event that the first coin is a head?

5. Are the events first coin a head, second coin a head, odd number of heads jointly independent?

6. Shuffle a deck of cards. Let A be the event that the first diamond comes before the first heart, B be the event that the first club comes before the first spade, and C the event that the first diamond comes before the first club. Is A independent of B? Is A independent of C? Are the three events jointly independent?

3 Binomial Distribution

Let's use independence to calculate probabilities involving a biased coin flip. (parenthetical question: why do we care about biased coin flips?)

A series of questions:

- If my coin has probability of heads p, what is the probability of tails?

- What is the probability I flip two heads in two flips?

- What is the probability I flip first a head then a tail?

- What is the probability I flip 4 heads then two tails (in that order)?

- Let's say I flip the biased coin 10 times. What is the sample space for this model? Are outcomes equally likely?

- How man sequences of 10 flips have exactly 6 heads and 4 tails?

- Pick one of the above sequences. What is the probability I get exactly that sequence with our biased coin?

- What is the probability I get exactly 6 heads in 10 flips?

- Generalize this: if I flip the biased coin n times, how many sequences of flips have exactly k heads?

- What is the probability of one such sequence?

- What is the probability of getting exactly k heads in n flips of the biased coin?

You've derived the binomial distribution!

4   Another distribution

Consider again our friend the biased coin, heads with probability p.

I want to flip it again and again until I get a head. (another parenthetical question: why would I care about this?)

Questions:

- What's the probability I flip 0 tails before I flip my first head?

- What's the probability I flip exactly 1 tail before my first head?

- Generalize this: what's the probability I flip exactly k tails before my first head?

- What's the sample space for this model?

- Specify the exact probability distribution. Have you seen it before?

conditional probability

Using the above model (biased coin with probability p, I flip until I get a head) answer the following:

- What is the conditional probability that I flip a total of 4 tails before the first head given that my first two flips were tails?

- Generalize this: write down the conditional probability function for the probability that I flip k tails before the first head given that my first t flips were tails.

- Does this probability function look familiar?

- draw some conclusions