

# Outcomes, Events, and Probabilities

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## 1 Outcomes and the Sample Space

The *sample space* of a probability model is an exhaustive, mutually exclusive set of outcomes of the model.

Exhaustive means that nothing else is possible in the model.

Mutually exclusive mean that the outcomes do not overlap.

Outcomes are the lowest level building blocks of your probability model. They cannot be divided any further.

We usually write down a sample space using set notation.

A natural sample space for rolling a die is  $\{1, 2, 3, 4, 5, 6\}$ , or  $\{1, \dots, 6\}$ .

A natural sample space for tossing a coin is  $\{Head, Tails\}$ .

Sample spaces can be more or less detailed. Two possible sample spaces for a model of tomorrow's weather are:

$$\{Rainy, Sunny, Snowy\}$$

or

$$\{(Rainy, Cold), (Rainy, Warm), (Sunny, Warm), (Sunny, Cold), (Snowy, Warm), (Snowy, Cold)\}$$

The second sample space is more detailed than the first, and so the probability model will be *finer*. The first model is *coarser*. Notice that a probability model can contain an outcome with 0 probability, like  $(snowy, warm)$ .

The set of outcomes can be finite, countably infinite, or uncountably infinite. Examples of each:

- finite:  $S = \{1, 2, 3, 4, 5, 6\}$
- countably infinite  $S = \{1, 2, 3, \dots\}$
- uncountably infinite: all real numbers between 0 and 1:  $S = [0, 1]$ .

To start we will consider finite and countably infinite sample spaces, which are called *discrete* probability models (as opposed to continuous probability models).

## 2 Events

*Events* are sets, or collections, of outcomes. An event can contain any number of outcomes. It can be a single outcome, 0 outcomes (the empty event), or all outcomes in the whole sample space.

All of the following are events in the model of rolling a single die ( $S = \{1, \dots, 6\}$ ):

- $E = \{1\}$ : you roll a 1
- $E = \{4, 5, 6\}$ : you roll a 4 or higher
- $E = \{1, 5\}$ : you roll a 1 or a 5
- $E = \{2, 3, 4, 5, 6\}$ : you don't roll a 1

### 3 Probabilities

A discrete probability model consists of a sample space and a probability function  $P$  that assigns a number to each outcome in the sample space. This function,  $P(x)$  follows two simple rules: a probability cannot be negative, and the sum of the probabilities of all the outcomes in the sample space must be 1.

Rules of discrete probability:

1. for every outcome  $x$ ,  $0 \leq P(x) \leq 1$ .
2. The probabilities of all outcomes sum to 1:  $\sum_{x \in S} P(x) = 1$
3. The probability of an event  $E$  is the sum of the probabilities of the outcomes in  $E$ :  $P(E) = \sum_{x \in E} P(x)$

General Axioms of Probability:

1.  $P(A) \in [0, 1]$  for all events  $A \subseteq S$
2.  $P(S) = 1$
3.  $P(A \cup B) = P(A) + P(B)$  if  $A \cap B = \emptyset$
4.  $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$  if  $E_i \cap E_j = \emptyset$  for all  $i \neq j$ .