Math 4221: Homework 3

Due 10/1/12

1

What is the probability that the SRW returns to 0 for the first time at step n? (find the asymptotics as $n \to \infty$)

2

A *cut* in a graph is a partition of the vertices into two sets. The *size* of a cut is the number of edges with one vertex in each side of the partition (the edges that are cut). For *any* graph with E edges, show that there is some cut of size at least $E/2$.

Bonus: Give an efficient algorithm to find a cut of size $\geq E/2$.

3

Show that if $p > (1+\epsilon)\log n/n$ then whp $G(n,p)$ has no isolated vertices.

4

Show that if $p < (1-\epsilon)\log n/n$ then whp $G(n,p)$ is not connected.

5

What is the probability that a SRW of 100 steps never goes above 10?

6

Let X be exponential with mean λ. What is $\mathbb{E}[X|X \geq 3]$?

7

Let $X_1, X_2, \ldots$ be iid exponential RV's of mean 1. Calculate the probability that $\sum_{i=1}^{k} X_i \leq 1$ but $\sum_{i=1}^{k+1} X_i > 1$.