Math 6221: Homework 4

Due March 12

1

Compute the characteristic function of a $N(0,1)$ random variable.

2

Let $Y_0 \sim Pois(\lambda)$, $Y_1 \sim Pois(Y_0), \ldots Y_n \sim Pois(Y_{n-1})$.

- Find the probability generating function of Y_0, Y_1, and Y_2.
- Using that, find the probability that $Y_n = 0$.

3

Let $U \sim Uniform[0,1]$ and let $B \sim Bin[n, U]$.

Find the distribution of B using generating functions.

4

Using characteristic functions, find a formula for $\mathbb{E}[X^k]$ when $X \sim N(0,1)$.

5

Let $p = c/n$ and X the number of triangles in the random graph $G(n,p)$. Show that as $n \to \infty$, X converges to a Poisson distribution.

6

Prove the Central Limit theorem assuming the X_i's are independent, with mean 0 and variance σ_i^2, where $1 \leq \sigma_i^2 < 100$ for all i.

7

Describe a statistical procedure to answer the following question:

In a large population of Zebras, each Zebra has two copies of a certain gene. Call these A_1 and A_2. When two Zebras mate, their offspring gets one copy from each parent, with each possible copy equally likely.

Given a sample of 100 Zebras, you do a genetic test and record the number of Zebras with each of the possible combinations: (A_1A_1), (A_1A_2), (A_2A_2).

Using this data how can you answer the question: Does this gene affect the Zebra's choice of partner?

In your answer include the null hypothesis, alternate hypothesis, test statistic, and method of calculating a p-value.

Then make up some data and test it.

8

Let $X_1, \ldots$ be iid with mean 0 and variance σ^2. For what choices of α can you prove that

$$U_n = \frac{X_1 + \cdots + X_n}{n^\alpha}$$

converges, and in what sense?

9

Explain how a casino would test whether its Roulette wheels are fair or not.

10

Let $X \sim Bin(1000, p)$. For what range of p is a Normal a better approximation to X than a Poisson, and vice versa?