Math 6221: Homework 5

Due March 28

1

Calculate the large deviation rate function for a Bernoulli(p) random variable. State the corresponding large deviation result for

$$\Pr[X_1 + \cdots + X_n > t]$$

where the X_i's are iid Ber(p).

2

Using the above, find the smallest $R(m,n)$ you can so that when we throw m balls into n bins uniformly at random, the maximum number of balls in a bin is $\leq R(m,n)$ whp.

3

Let $p = cn^{-2/3}$. Let X be the number of vertices of $G(n,p)$ that are *not* in a triangle. Calculate $\mathbb{E}X$.

4

Consider a branching process with offspring distribution $Pois(\lambda)$. Assume $\lambda > 1$. Find a formula for the probability that the branching process goes extinct.

5

Using the formula above, let $\lambda = 1 + \epsilon$ and find the asymptotics of the survival probability of the branching process as $\epsilon \to 0$.

6

Find upper and lower bounds on the probability that the sum of n independent $Pois(\lambda)$ random variables is greater that $(\lambda + \epsilon)n$.

7

Let Z_n be the number of individuals at step n of a branching process, and let p_e be the extinction probability of the branching process.

Show that $M(n) = p_e^{Z_n}$ is a Martingale.

Does $M(n)$ converge to some random variable? If so, in what sense and to what?

8

Let S_n be a simple random walk with bias p. Let $M_n = (q/p)^{S_n}$ Show that M_n is a Martingale and show that it converges almost surely.

9

Prove that simple symmetric random walk in dimension 3 or higher is transient.

10

Is a branching process transient or recurrent? Why?