FRONTIERS OF STATISTICAL MECHANICS AND THEORETICAL COMPUTER SCIENCE
2021

All talks on Zoom, all times listed are UTC time.

Monday, December 13

15:00 - 15:40 UTC: *Recent progress on random field Ising model*

Jian Ding

Random field Ising model is a canonical example to study the effect of disorder on long range order. In 70's, Imry-Ma predicted that in the presence of weak disorder, the long-range order persists at low temperatures in three dimensions and above but disappears in two dimensions. In this talk, I will review mathematical development surrounding this prediction, and I will focus on recent progress on exponential decay (joint with Jiaming Xia) and correlation length in dimension two (joint with Mateo Wirth), a new proof for long range order in dimension three (joint with Zijie Zhuang) and a recent general inequality for the Ising model (joint with Jian Song and Rongfeng Sun) which has implications for random field Ising model.

15:50 - 16:30 UTC: *The upper tail for triangles in random graphs*

Wojciech Samotij

Let X denote the number of triangles in the random graph $G_{n,p}$. The problem of determining the asymptotics of the logarithimic upper tail probability of X, that is, $\log \Pr(X > (1+\delta)\mathbb{E}[X])$, for every fixed positive δ has attracted considerable attention of both the combinatorics and the probability communities. We shall present an elementary solution to this problem, obtained recently in a joint work with Matan Harel and Frank Mousset. The crux of our approach is a simple probabilistic argument, inspired by the work of Janson, Oleszkiewicz and Ruciński, that reduces the estimation of this upper tail probability to a counting problem.

16:40 - 17:20 UTC: *In Hopes of Systematizing Information-Computation Gaps*

Tselil Schramm

Many problems in statistics exhibit an information-computation gap: even when the problem is information-theoretically solvable, we do not know polynomial-time algorithms. In this survey-style talk I will describe a recent-ish line of work which aims to study this phenomenon systematically, by establishing a universality class of problems which exhibit information-computation gaps in powerful restricted models of computation including sum-of-squares semidefinite programs, statistical query algorithms, and more.

Tuesday, December 14

15:00 - 15:30 UTC: *Continuous symmetry breaking along the Nishimori line*

Christophe Garban

In this talk, I will start by introducing spin systems on $\mathbb{Z}^d$ with continuous symmetry and I will discuss a new way to prove continuous symmetry breaking for any dimension $d \geq 3$ which does not rely on the standard tool for this known as 'reflection positivity'. Our method applies to models whose spins take values in S^1, $SU(n)$ or $SO(n)$ in the presence of a certain quenched disorder called the Nishimori line. The proof of continuous symmetry breaking is based on two ingredients:

(1) the notion of 'group synchronization' in Bayesian statistics. In particular a recent result by Abbe, Massoulié, Montanari, Sly and Srivastava (2018) which proves group synchronization when $d \geq 3$.
(2) a gauge transformation on both the disorder and the spin configurations which goes back to Nishimori (1981).

I will end the talk with an application of these techniques to a deconfining transition for $U(1)$ lattice gauge theory on the Nishimori line. This is a joint work with Tom Spencer (https://arxiv.org/abs/2109.01617).

15:40 - 16:10 UTC: *Sampling Matrices from HCIZ Densities*

Jonathan Leake

Log-linear density functions on unitary conjugation orbits of Hermitian matrices (with respect to the induced Haar measure) are important in various settings in physics, random matrix theory, statistics, and machine learning. Such densities are known as Harish-Chandra–Itzykson–Zuber (HCIZ) densities, and until recently the general question of efficient sampling according to such densities remained open. In this talk, we describe an efficient sampling algorithm which exploits a strong connection between a given unitary conjugation orbit and an associated polytope, called its Gelfand-Tsetlin polytope. More conceptually, we give an interesting example of how one can sample from a non-convex (highly symmetric) manifold by sampling from a convex polytope. This is based on joint work with Colin McSwiggen and Nisheeth Vishnoi.

16:20 - 16:50 UTC: *Changing the Temperature for Algorithm Design*

Holden Lee

A key problem in computer science, statistics, and statistical mechanics is to sample from a probability distribution given access to its density function (up to normalization constant). Markov Chain Monte Carlo is the predominant approach; however, local Markov chains are known to mix slowly for multimodal distributions. The physics-inspired idea of "changing the temperature" has led to a wealth of heuristic algorithms, such as simulated annealing, simulated tempering, and parallel tempering, to speed up sampling and optimization–however, our theoretical understanding of these methods is still limited. I'll show that for simple multimodal distributions, simulated tempering can speed up the mixing time from exponential to polynomial. I'll introduce the "two-scale functional inequalities" that are key to the analysis, and explore the further potential of these ideas for algorithm design.

17:00 - 17:30 UTC: *Low-temperature Ising dynamics from random initializations*

Reza Gheissari

Local-update Markov chains (e.g., the Glauber dynamics) for spin systems like the Ising model are well-known to undergo exponential slowdowns at low temperatures. This is due to a bottleneck between the two phases of the model, one where most spins are plus, and one where most spins are minus. It is a folklore belief that the Glauber dynamics should be fast to equilibrate if they are initialized from a randomly chosen ground state, i.e., with probability $\frac{1}{2}$ the all-plus configuration and with probability $\frac{1}{2}$ the all-minus configuration. I will describe joint work with Alistair Sinclair making progress towards this conjectured behavior for the Ising model on $\mathbb{Z}^d$ throughout its low-temperature regime.

Wednesday, December 15

15:00 - 15:30 UTC: *Approximate counting and sampling via local central limit theorems*

Vishesh Jain

We introduce a framework for exploiting local central limit theorems as algorithmic tools. As applications, for general graphs G with bounded maximum degree Δ, we provide a deterministic fully polynomial time approximation scheme (FPTAS) (respectively, a quasi-linear time randomized algorithm) to count (respectively, sample from the uniform distribution on): (i) matchings of a given size k, for all $k \leq (1-\delta)m^*(G)$, for arbitrary $\delta > 0$, where $m^*(G)$ is the matching number of G. (ii) independent sets of a given size k, for all $k \leq (1-\delta)\alpha(\Delta)$, for arbitrary $\delta > 0$, where $\alpha(\Delta)$ is the NP-hardness threshold for this problem.

Joint work with Will Perkins, Ashwin Sah, and Mehtaab Sawhney.

15:40 - 16:10 UTC: *Independent sets in random subgraphs of the hypercube*

Gal Kronenberg

The number of independent sets in the hypercube $\{0,1\}^d$ was estimated precisely by Korshunov and Sapozhenko in the 1980s and recently refined by Jenssen and Perkins. In this talk we will discuss new results on the number of independent sets in a random subgraph of the hypercube. The results extend to the hardcore model and rely on an analysis of the antiferromagnetic Ising model on the hypercube. This talk is based on joint work with Yinon Spinka.

16:20 - 16:50 UTC: *Mixing in frustrated systems at low temperatures*

Mark Jerrum

Line graphs — of which the kagome and pyrochlore lattices are examples — are well studied in graph theory. I'll start with a result obtained jointly with Martin Dyer, Marc Heinrich and Haiko Müller concerning the antiferromagnetic Ising model on a line graph G. The informal statement is that Glauber dynamics mixes in polynomial time (in the number of vertices in G) at any non-zero temperature. The main tool used in establishing this result is the canonical paths method, specifically the 'winding' technology of McQuillan. I'll then describe subsequent work by others in this direction, based on more recent techniques such as interpolation along lines in zero-free regions of the partition function, and spectral independence. The phenomenon of mixing at all temperatures is intriguing and seems worth studying further.

I'll finish with some rather vague remarks about the limitations of our current techniques when dealing with models that exhibit weak spatial mixing but not strong spatial mixing.

17:00 - 17:30 UTC: *Generalized Ising perceptron models*

Nike Sun

The perceptron is a toy model of a single-layer neural network that 'stores' a collection of given patterns. We consider the model on the N-dimensional discrete cube with $M = N * \alpha$ patterns, with a general activation function U that is bounded above. For U bounded away from zero, or U a one-sided threshold function, it was shown by Talagrand (2000, 2011) that for small alpha, the free energy of the model converges in the large-N limit to the replica symmetric formula conjectured in the physics literature (Krauth and Mezard, 1989). We give a new proof of this result, which covers the more general class of all functions U that are bounded above and satisfy a certain variance bound. Based on joint work with Erwin Bolthausen, Shuta Nakajima, and Changji Xu; as well as an earlier work with Jian Ding.