

# MATH 215: Introduction to Advanced Mathematics

## Homework 2

Due in class, Friday September 14

- (1) Prove the following statements.
  - (a) For integers  $a$  and  $b$ ,  $a$  divides  $b$  implies  $a$  divides  $-b$ .
  - (b) For real numbers  $x, y$ , and  $z$ ,  $x^2 + y^2 + z^2 \geq xy + xz + yz$ .
  - (c) There do not exist integers  $a$  and  $b$  so that  $9a + 6b = 44$ .
- (2) Prove that for real numbers  $a, b$ ,

$$|a + b| \leq |a| + |b|.$$

Give a necessary and sufficient condition for equality.

- (3) Prove that for all integers  $n$ ,  $n^2$  is odd  $\Rightarrow n$  is odd. (Hint: use proof by contrapositive).
- (4) Prove that for real numbers  $a$  and  $b$ ,  $ab = 0 \Leftrightarrow a = 0$  or  $b = 0$ .
- (5) Prove that for real numbers  $x$ , if  $x^4 \geq 8x$  then  $x \leq 0$  or  $x \geq 2$ .