

MATH 215: Introduction to Advanced Mathematics

Homework 3

Due in class, Friday September 21

- (1) Prove by induction that $\sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n .
- (2) Define the sequence a_n by the following:
 - $a_1 = 1$
 - $a_{k+1} = \frac{6a_k+5}{a_k+2}$ for every positive integer k .

Prove by induction that $0 < a_n < 5$ for all positive integers n .

- (3) Prove that

$$(1+x)^n \geq 1+nx$$

for all positive integers n and all real numbers $x > -1$.

- (4) Prove that $n^3 - 3n^2 - n$ is divisible by 3 for all integers n (both positive and negative).