

# MATH 215: Introduction to Advanced Mathematics

## Homework 4

Due in class, Friday September 28

- (1) Let  $A = [0, 2]$ ,  $B = (1, 3)$  and  $C = (-\infty, 2)$ . Compute the following:
- (a)  $A \cap B$
  - (b)  $A \cap C$
  - (c)  $C - A$
  - (d)  $A - C$
  - (e)  $C^c$  (with respect to the universe  $\mathbb{R}$ )
  - (f)  $A \cup B$
  - (g)  $B \cup C$
- (2) Determine whether the following statements are true or false. If true, prove the statement. If false, first write the negation of the statement and then disprove the original statement.
- (a)  $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, y^2 = x$ .
  - (b)  $\forall x \in \mathbb{R} - \{0\}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, xz = y$ .
  - (c)  $\exists x \in (0, 1), \forall y \in (0, 1), \exists z \in (0, 1), x + y + z \geq 2$ .
- (3) Using a truth table prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- (4) Determine the following set (i.e. write out all of its elements):

$$A = \{a \in \mathbb{Z} - \{7\} | 20 < a^2 < 70\}$$