UNIQUENESS METHODS IN STATISTICAL MECHANICS: RECENT DEVELOPMENTS AND ALGORITHMIC APPLICATIONS

Monday, December 14

15:00 UTC: *Statistical mechanics and graph generating functions*

David Brydges

In 1940 Joseph Edward and Maria Goeppert Mayer published their influential book "Statistical Mechanics" which included beautiful theorems about corrections to the ideal gas law ($PV \propto T$) in terms of generating functions for three classes of graphs. Furthermore they found surprising simple functional relations between these generating functions. I will review this background and some of the later development to set the stage for new progress to be presented by other speakers.

15:50 UTC: *Introduction to approximate counting and sampling*

Eric Vigoda

I will give an introduction to approximate counting and sampling techniques. I will focus on the following topics:

- Counting and sampling connection
- Path coupling and Dobrushin uniqueness
- Strong Spatial Mixing and fast mixing (on the grid and amenable graphs)
- Weitz's SAW tree and correlation decay
- Spectral independence

16:40 UTC: *On the zeros and approximation of the Ising partition function*

Alexander Barvinok

We consider the problem of efficient computation (approximation) of the sum of $\exp f(x)$, where x ranges over the Boolean cube $\{-1,1\}^n$ and f is a quadratic or cubic polynomial. We prove that the sum can approximated in quasi-polynomial $n^{O(\log n)}$ time as long as the Lipschitz constant in the L^1 metric of the non-linear part of f is strictly smaller than 1. The algorithm is supplied by the interpolation method, for which we prove that the sum remains non-zero, as long as f is replaced by a complex polynomial, sufficiently close to f. The bounds are asymptotically optimal. This is a joint work with N. Barvinok.

17:30 UTC: Informal discussion on Gather.town

Tuesday, December 15

15:00 UTC: *Metastability for the dilute Curie–Weiss model with Glauber dynamics*

Elena Pulvirenti

We analyse the metastable behaviour of the dilute Curie–Weiss model subject to a Glauber dynamics. The model is a random version of a mean-field Ising model, where the coupling coefficients are replaced by i.i.d. random coefficients, e.g. Bernoulli random variables with fixed parameter p. This model can be also viewed as an Ising model on the Erdos–Renyi random graph with edge probability p. The system is a Markov chain where spins flip according to a Metropolis dynamics at inverse temperature β. We compute the average time the system takes to reach the stable phase when it starts from a certain probability distribution on the metastable state (called the last-exit biased distribution), in the regime where the system size goes to infinity, the inverse temperature is larger than 1 and the magnetic field is positive and small enough. We obtain asymptotic bounds on the probability of the event that the mean metastable hitting time is approximated by that of the Curie–Weiss model. The proof uses the potential theoretic approach to metastability and concentration of measure inequalities. This is a joint work with Anton Bovier and Saeda Marello.

15:40 UTC: *Algorithmic and combinatorial applications of the cluster expansion*

Matthew Jenssen

The cluster expansion is a classical tool from statistical physics traditionally used to study the phase diagram of lattice spin models. Recently, the cluster expansion has enjoyed a number of applications in two new contexts: i) the design of efficient approximate counting and sampling algorithms for spin models on graphs and ii) classical enumeration problems in combinatorics. In this talk, I'll give an introduction to the cluster expansion and discuss some of these recent developments.

16:20 UTC: *Crystalline ordering in hard-core lattice particle systems*

Ian Jauslin

I will present a class of hard-core lattice particle systems which exhibit a crystalline phase at high densities. The key ingredient of the proof is to show that the Gaunt-Fisher high-fugacity expansion is convergent for such models, which we accomplish using methods from Pirogov-Sinai theory. Crystallization can then be proved by studying the lower order terms of the expansion and bounding the remainder. This is joint work with Joel L. Lebowitz.

17:00 UTC: *Fractionally Log-Concave and Sector-Stable Polynomials: Counting Planar Matchings and More*

Nima Anari

We show fully polynomial time randomized approximation schemes for counting matchings of a given size, or more generally sampling/counting monomer-dimer systems in planar, not-necessarily-bipartite, graphs. While perfect matchings on planar graphs can be counted exactly in polynomial time, counting non-perfect matchings was shown by Jerrum [Jer87] to be #P-hard, who also raised the question of whether efficient approximate counting is possible. We answer this affirmatively by showing that the multi-site Glauber dynamics on the set of monomers in a monomer-dimer system always mixes rapidly, and that this dynamics can be implemented efficiently on families of graphs where counting perfect matchings is tractable.

In order to analyze mixing properties of the multi-site Glauber dynamics, we establish two new notions for generating polynomials of discrete set-valued distributions: sector-stability and fractional log-concavity. These notions generalize well-studied properties like real-stability and log-concavity, but unlike them robustly degrade under useful transformations applied to the distribution. We relate these notions to pairwise correlations in the underlying distribution and the notion of spectral independence introduced by Anari, Liu, and Oveis Gharan [ALO20], providing a new tool for establishing spectral independence based on zeros of polynomials. As a byproduct of our techniques, we show that polynomials whose roots avoid certain regions of the complex plane must satisfy a form of convexity, extending a classic result established between hyperbolic and log-concave polynomials by Gårding [Går59] to new classes of polynomials.

As further applications of our results, we show how to sample efficiently from certain partition-constrained strongly Rayleigh distributions, and certain asymmetric determinental point processes using natural Markov chains.

Joint work with Yeganeh Alimohammadi, Kirankumar Shiragur, and Thuy-Duong Vuong.

17:40 UTC: Informal discussion on Gather.town

Wednesday, December 16

15:00 UTC: *Revisiting Groeneveld's approach to the virial expansion*

Sabine Jansen

A generalized version of Groeneveld's convergence criterion for the virial expansion and generating functionals for weighted 2-connected graphs is proven. The criterion works for inhomogeneous systems and yields bounds for the density expansions of the correlation functions ρ_S (aka distribution functions or factorial moment measures) of grand-canonical Gibbs measures with pairwise interactions. The proof is based on recurrence relations for graph weights related to the Kirkwood-Salsburg integral equation for correlation functions. The proof does not use an inversion of the density-activity expansion, however a Moebius inversion on the lattice of set partitions enters the derivation of the recurrence relations. Based on arXiv:2009.09211 [math-ph].

15:40 UTC: *Analyticity for Classical Gasses via Recursion*

Marcus Michelen

Inspired by techniques from theoretical computer science, we turn to a classical question in statistical physics: for what range of parameters do classical particles in the continuum interacting via a pair potential remain in a gaseous state? Or in other words, for which parameters is the infinite volume pressure analytic? The classical approaches to this question—such as understanding the convergence of the cluster expansion or the Kirkwood-Salsburg equations—may be limited by the presence of singularities of the pressure away from the positive real axis in the complex plane. In the case of a repulsive pair potential, we prove analyticity of the pressure in a range of activities that improves the classical bound on absence of phase transition by a factor e^2. Joint work with Will Perkins.

16:20 UTC: *Optimal Mixing of Glauber Dynamics: Entropy Factorization via High-Dimensional Expansion*

Zongchen Chen

We consider the Glauber dynamics (also called Gibbs sampling) for sampling from a discrete high-dimensional space, where in each step one variable is chosen uniformly at random and gets updated conditional on all other variables. We show an optimal mixing time bound for the Glauber dynamics in a variety of settings. Our work presents an improved version of the spectral independence approach of Anari et al. (2020) and shows $O(n \log n)$ mixing time for graphical models/spin systems on any n-vertex graph of bounded degree when the maximum eigenvalue of an associated influence matrix is bounded. Our proof approach combines classic tools of entropy tensorization/factorization and recent developments of high-dimensional expanders.

As an application of our results, for the hard-core model on independent sets weighted by a fugacity λ, we establish $O(n \log n)$ mixing time for the Glauber dynamics on any n-vertex graph of constant maximum degree D when $\lambda < \lambda_c(D)$ where $\lambda_c(D)$ is the critical point for the uniqueness/non-uniqueness phase transition on the D-regular tree. More generally, for any antiferromagnetic 2-spin system (e.g., Ising model) we prove $O(n \log n)$ mixing time of the Glauber dynamics on any bounded degree graph in the corresponding tree uniqueness region. Our results apply more broadly; for example, we also obtain $O(n \log n)$ mixing for sampling random q-colorings of triangle-free graphs of maximum degree D when the number of colors satisfies $q > aD$ where $a = 1.763\ldots$, and $O(m \log n)$ mixing for generating random matchings of any graph with bounded degree and m edges.

17:00 UTC: *Phase transitions and finitary codings*

Yinon Spinka

In this talk we will explore the connection between the two seemingly unrelated concepts appearing in the title. Finitary coding is an ergodic-theoretic notion which is concerned with the expressibility of certain processes in terms of other processes. We will focus on the question of whether or not a given system admits a finitary coding from an i.i.d. process. Such a coding may be seen as an algorithm for exact sampling from an infinite-volume Gibbs measure. After introducing the notion of finitary coding, we will discuss some results which establish close links between it and phase transitions.

17:40 UTC: Informal discussion on Gather.town