Math 6221: Homework 6

Due April 10

1

Let $\mathcal{F}_n$ be a filtration and let the filtration $\mathcal{G}_m$ be a subsequence of $\mathcal{F}_n$. Show that if $(X_n, \mathcal{F}_n)$ is a martingale, then $(X_m, \mathcal{G}_m)$ is a martingale.

2

The Urn Problem: start with one red ball and one green. At each step pick a ball at random, and return it along with an additional ball of the same color. Let X_n be the proportion of red balls. To what and in what sense does X_n converge?

3

Prove that a Doob's Martingale, $X_n = \mathbb{E}[X|\mathcal{F}_n]$ converges almost surely to X.

4

Prove an isoperimetric inequality for S_n, the space of permutations on n elements, with the distance function defined by the number of elements sent to different places by two permutations.

5

Consider a random graph with the vertex exposure filtration. Let X be the number of triangles in the graph $G(n, p)$. What is $\mathbb{E}[X|\mathcal{F}_3]$? Same question if $\mathcal{F}_n$ is the edge exposure filtration.

6

Use Azuma's inequality to prove a concentration result for the number of empty bins when you thrown m balls at random into n bins.