

Lecture 1: Introduction and Probability Spaces

Will Perkins

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- ① Go over course mechanics and material
- ② Fill out a course survey
- ③ Look ahead at course themes and the two main theorems
- ④ Sigma Fields and Probability Spaces

① Mathematical Foundations of Probability Theory

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- 2 Asymptotic Perspective

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- ④ Tools and Methods
- ⑤ Probabilistic Models

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- ② Central Limit Theorem

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- 3 Are their proofs important or useful?

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- 4 Computer Science: Randomness as an algorithmic tool. Connection between randomness and computational complexity.
- 5 Pure Math: probabilistic method in combinatorics and number theory.