

MATH 215: Introduction to Advanced Mathematics

Homework 1

Due in class, Friday September 7

- (1) Write the truth table for the following statements
 - (a) $\text{not}(P \vee Q)$
 - (b) $P \vee (\text{not}P \wedge Q)$
 - (c) $((P \Rightarrow Q) \wedge (Q \Rightarrow R) \wedge (P \vee Q)) \Rightarrow R$
- (2) Using truth tables prove the following statements (these are known as DeMorgan's Laws):
 - (a) $\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q)$
 - (b) $\neg(P \wedge Q) \Leftrightarrow (\neg P) \vee (\neg Q)$
- (3) Using truth tables prove the following three statements are equivalent:
 - $P \Rightarrow Q$
 - $(P \text{ or } Q) \Leftrightarrow Q$
 - $(P \text{ and } Q) \Leftrightarrow P$
- (4) For this question we will work with the integers, and assume the basic properties of addition, subtraction, multiplication and ordering (e.g. $0 < 1$, $a + b = b + a$ etc.) Do not use division or fractions. Recall the definition: For integers a, b we say a divides b if there exists an integer c so that $ac = b$. Prove or disprove the following statements.
 - (a) 5 does not divide 7.
 - (b) If a divides b and a divides c then a divides $(b + c)$.
 - (c) If a divides b and a divides c then b divides c .
 - (d) If 4 divides a then 8 divides a .
 - (e) 1 divides every integer a .
 - (f) If 3 divides a and 5 divides b then 15 divides ab .